Properties of Functions

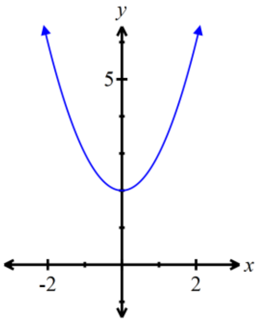
Even and Odd Functions: The words **even** and **odd**, when applied to a function f, describe the symmetry that exists for the graph of the function.

Even Function: A function f is even if, for every number x in its domain, the number $-x$ is also in the domain and $f(-x) = f(x)$. Even functions are **symmetric with respect to the *y*-axis.**

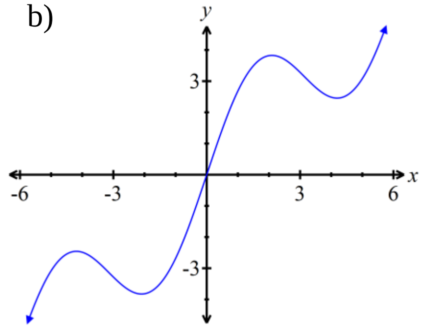
Odd Function: A function f is odd if, for every number x in its domain, the number $-x$ is also in the domain and $f(-x) = -f(x)$. Odd functions are **symmetric with respect to the origin.**

Examples: Determine whether each graph is an even function, odd function, or neither.

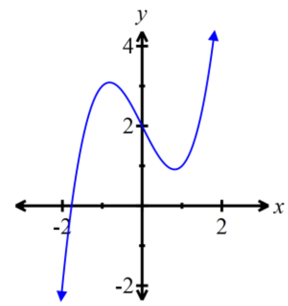
a)



b)



c)



Examples: Determine whether each of the following functions is even, odd, or neither. Then determine whether the graph is symmetric with respect to the *y*-axis, the origin, or neither.

a) $f(x) = x^2 - 2$

b) $f(x) = 4x^3 + x^2 - 1$

c) $f(x) = x^3 + x$

d) $f(x) = |x| + 5$

Increasing, Decreasing, and Constant Graphs: If you look from left to right along the graph of the function, you will notice parts are *rising*, parts are *falling* and parts are *horizontal*. In such cases, the function is described as *increasing*, *decreasing*, or *constant*, respectively.

Definitions:

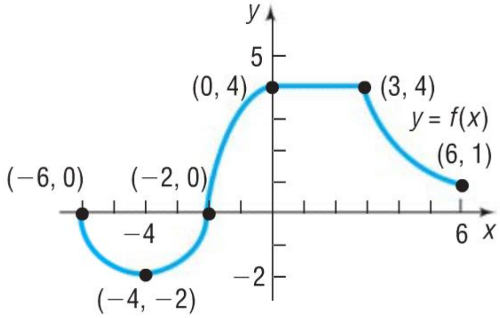
A function f is ***increasing*** if for any choice of x_1 and x_2, where $x_1 < x_2$, then $f(x_1) < f(x_2)$.

A function f is ***decreasing*** if for any choice of x_1 and x_2, where $x_1 < x_2$, then $f(x_1) > f(x_2)$.

A function f is ***constant*** if for any choice of x_1 and x_2, where $x_1 < x_2$, then $f(x_1) = f(x_2)$.

Increasing **Decreasing** **Constant**

Example: Determine where the graph is increasing, decreasing, or constant.

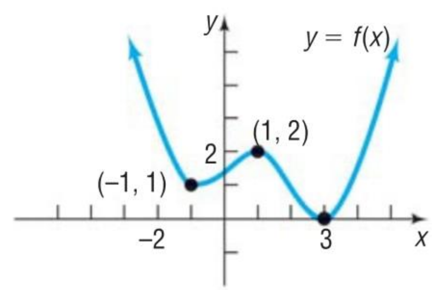


Local Maxima and Minima:
When a graph is increasing to the left of a point on the graph, and decreasing to the right of that point on the graph, then the value is a ***local maximum*.** When a graph is decreasing to the left of a point on the graph, and increasing to the right of that point on the graph, then the value is a ***local minimum*.**

Local Maximum **Local Minimum**

- ★ **Note:** If a question asks "Where…", "On what interval(s)…", or "At what number(s)…", it is asking for *x*-coordinates. If it asks "What is..." or "Find the value of…", it is asking for a *y*-coordinate.

Example:



a) At what number(s), if any, does f have a local maximum?

b) What are the local maxima?

c) At what number(s), if any, does f have a local minimum?

d) What are the local minima?

e) List the intervals where f is increasing and the intervals where f is decreasing.

Using a Graphing Calculator to Find Local Minima and Maxima
To find the exact value at which a function f has a local maximum or local minimum usually requires calculus. However, a graphing calculator may be used to approximate these values.

1. Press Y = and enter function.
2. Press GRAPH.
3. Enter the domain by pressing WINDOW. Xmin = smallest #, Xmax = largest #.
4. Press GRAPH.
5. Press ZOOM, and choose 0 : ZoomFit.
6. Press 2ND TRACE (CALC) and choose 3 : minimum or 4 : maximum, enter.
7. Move the arrows until you are left of the minimum or maximum and press enter.
8. Move the arrows until you are right of the minimum or maximum and press enter.
9. Move the arrows until you are near the minimum or maximum and press enter.
10. The Y= ___ on the bottom right gives the minimum or maximum.

Example: Use a graphing calculator to graph $f(x)=x^3-3x+2$ for $-2<x<2$. Approximate where f has a local maximum and where f has a local minimum. Also find the values of the minimum and maximum.

Find the Average Rate of Change of a Function

To find the average rate of change of a function between any two points on its graph, calculate the slope of the line containing the two points.

If *a* and *b*, $a \neq b$, are in the domain of a function $y = f(x)$, the **average rate of change of *f*** from *a* to *b* is defined as:

Average rate of change = $\dfrac{\Delta y}{\Delta x} = \dfrac{f(b) - f(a)}{b - a} \quad a \neq b$

The symbol Δy above is "the change in *y*," and Δx is the "change in *x*." The average rate of change of *f* is the change in *y* divided by the change in *x*.

Example:

Find the average rate of change of $f(x) = 3x^2$ for the following intervals:

a) From 1 to 3 or [1, 3]

b) From 1 to 5 or [1, 5]

c) From 1 to 7 or [1, 7]